Vol. 48 (2001) *REPORTS ON MATHEMATICAL PHYSICS* No. 3

SPACE-TIME GEOMETRY WITH NONHOLONOMIC TIME FIELD

V. I. BASHKOV and YU. V. SINTSOVA

Kazan State University, Physical Department, Kremlevskaya 18, Kazan 420008, Russia
(e-mail: Victor.Bashkov@ksu.ru)

(Received November 6, 2000 – Revised version May 8, 20001)

The aim of this paper is to demonstrate an effect of time nonholonomity which appears in accelerated reference systems. We suppose that an accelerated reference system is closely related to a physical field which changes the space-time geometry. On the space-time $\widehat{M} = M \times R$ we take a metric $\widehat{g}$ of general form which is invariant under time shifts $t \to t + t_0$, only. The physical field is described by a 1-form θ and a function ϕ given on a spatial manifold M.

These considerations are motivated by the model suggested in [1, 2] which interprets the Sagnac effect [4, 5] as an effect caused by a deformation of space-time geometry generated by the disk rotation. We demonstrate that this effect occurs for our general space-time metric. To this end, we consider the space-time $\widehat{M}$ as a principal bundle $\widehat{M} \to M$ with group R, and use the fact that the distribution H orthogonal to the fibres gives an infinitesimal connection in this bundle. We note that there arises the same effect, called a generalized Sagnac effect, and prove that this one is determined by the holonomy of connection H, i.e. it occurs because H is not integrable.

1. Introduction

The aim of the present paper is to demonstrate an effect of time nonholonomity which appears in accelerated reference systems. We suppose that an accelerated reference system is closely related to a physical field which changes the space-time geometry. Let M be a smooth manifold equipped with a metric q, and $\widehat{M} = M \times R$ be the space-time. On $\widehat{M}$ we take the metric

$$\widehat{g}_{(p,t)}(X, Y) = e^{\phi(p)}dt(X)dt(Y) + \theta_p(\pi_*X)dt(Y) + +\theta_p(\pi_*Y)dt(X) - q_p(\pi_*X, \pi_*Y), \tag{1}$$

where $\pi_* : T_{(p,t)}\widehat{M} \to T_pM$ is the differential of the projection $\pi : \widehat{M} \to M$, $(p, t) \to p$, $\phi : M \to R$ is a function, and θ is a 1-form on M.

Note that $\widehat{g}$ contains nondiagonal terms $dtdx$, as it usually occurs for the accelerated systems, and $\widehat{g}$ is the most general metric on $\widehat{M}$ invariant under the time shifts $t \to t + t_0$. We assume that the physical field which deformes the space-time geometry is described by θ and ϕ. For $M = R^3$ and $q = dx^2 + dy^2 + dz^2$, if $\theta = 0$ and $\phi = 0$, the metric $\widehat{g}$ turns into the Minkowskian metric.

These considerations are motivated by a model suggested in [1, 2] which interprets the Sagnac effect [4, 5] as an effect caused by a space-time geometry defor-